LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – APRIL 2010

ST 1814 / 1809 - MEASURE AND PROBABILITY

Date & Time: 27/04/2010 / 1:00 - 4:00   Dept. No.   Max. : 100 Marks

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SECTION –A

Answer all the questions. Each question carries TWO marks (10 x 2 = 20)

1. Define monotone class of sets.

2. Find the Lebesgue measure of
   (i) {2,3} and (ii) {0,10]U(6,11).

3. Define the integral of a non- negative measurable function with respect to a measure.

4. If f and g are measurable, examine whether max{f,g} is measurable.

5. Show that $\left| \int_{\Omega} X \, d\mu \right| \leq \int_{\Omega} |X| \, d\mu$.

6. Define a random variable and its probability distribution.

7. If X is a random variable with continuous distribution function F, obtain the probability distribution of F(X).

8. If X is a random variable with $P[X=(-1)^k 2^k] = 1/2^k$, k = 1,2,3…, examine whether E(X) exists.

9. If ϕ_1 and ϕ_2 are characteristic functions (CF), show that $\phi_1 . \phi_2$ is a CF.

10. State Lindeberg – Feller central limit theorem

SECTION-B (5×8 =40 marks)

Answer any FIVE questions. Each question carries EIGHT marks.

11. (a) Define limit inf A_n and limit sup A_n of a sequence of sets.
    (b) For a sequence $\{A_n\}$ of sets, if $A_n \to A$, show that $A_n^c \to A^c$. (4+4 marks)

12. Prove that a non-negative Borel measurable function is the limit of a non-decreasing sequence of non-negative finite valued simple functions.

13. State and prove Monotone convergence theorem.

14. Let μ be a finitely additive set function on the field $\Im$. Show that
    (i) $\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$ for every A, B ∈ $\Im$.
    (ii) A, B ∈ $\Im$, $A \subset B \Rightarrow \mu(A) \leq \mu(B)$.

15. If X and Y are independent, show that the characteristic function of (X+Y) is the product of their characteristic functions. Is the converse true? Justify.

16. Define (i) convergence in probability.
    (ii) almost sure convergence for a sequence of random variables.
    Show that convergence in quadratic mean implies the convergence in probability.

17. State and prove Kolmogorov zero-one law for a sequence of independent random variables.

18. In the usual notation, prove that

$$\sum_{n=1}^{\infty} P[|X| \geq n] \leq E|X| \leq 1 + \sum_{n=1}^{\infty} P[|X| \geq n].$$

SECTION-C (2 x 20 = 40 marks)
Answer any two questions. Each question carries TWENTY marks.

19. (a) Define the limit of a sequence of sets.
(b) If {An, n≥1} is any sequence of sets, show that there exists a sequence { Bn, n≥1} of disjoint sets such that $\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} B_n$.
(c) Show that a σ-field is a monotone field and conversely (4+8+8)

20. (a) State and prove Lebesgue dominated convegence theorem. (8 marks)
(b) State and prove Fatou's lemma. (12 marks)

21. (a) State and prove Borel zero –one law
(b) If $\{X_n , n \geq 1\}$is a sequence of independent and identically distributed random variables with common frequency function e^{-x} , $x \geq 0$, prove that

$$P\left[\overline{\lim}\left(\frac{X_n}{\log n}\right) > 1\right] = 1.$$ (12+8 marks)

22. (a) State and prove Levy continuity theorem for a sequence of characteristic functions. (12 marks).
(b) Show that Liapounov theorem is a particular case of Lindeberg-Feller central limit theorem. (8 marks)

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